Comparative Statics Exercises

1) Prove that if f is differentiable, then f has increasing differences if and only if $\frac{\partial^2 f(x,\theta)}{\partial x \partial \theta} \geq 0$.

2) Consider the parameterized optimization problem

$$\max_{x \in [0,1]} \quad (1-x)p(x) + q(1-p(x))$$

where $p(\cdot) > 0$ is strictly increasing and concave. Assume that x^* is on the interior of $[0,1]$.

i) Use the implicit function theorem to show how the optimal choice of x given q, $x^*(q)$ changes as q changes.

ii) How does the value function change as q changes?

3) Consider the parameterized optimization problem

$$\max_{x,z} \quad f(x,z;\theta)$$

where $x, y, \theta \in \mathbb{R}$. Assume f is twice continuously differentiable. Let f_{ij} denote $\frac{\partial^2 f}{\partial i \partial j}$ for $i, j \in \{x, z, \theta\}$ i.e. f_{xx} is the second derivative of f and f_{xz} is the cross partial derivative of f with respect to x and z. Let $(x^*(\theta), z^*(\theta))$ be a solution.

i) What conditions on f_{xx}, f_{zz}, and f_{xz} must hold for $(x^*(\theta), z^*(\theta))$ to be a local maximum? (Hint: what must be true of the Hessian matrix with respect to choice variables at a local maximum?)

ii) Use the implicit function theorem to characterize $\frac{\partial}{\partial \theta} x^*(\theta)$ and $\frac{\partial}{\partial \theta} z^*(\theta)$ in terms of f_{ij}.

iii) Let $f_{x\theta} = 0$, $fz\theta < 0$, $f_{xz} > 0$. Describe the comparative statics. Now let $f_{xz} < 0$

and describe the comparative statics. Interpret this result.

iv) Show that if f is supermodular, then $\frac{\partial}{\partial\theta}x^*(\theta) > 0$ and $\frac{\partial}{\partial\theta}z^*(\theta) > 0$.