Linear Spaces Exercises

1) Let S be a basis for X so that for every $x \in X$, there exist elements $x_1, x_2, ..., x_n \in X$ and scalars $\alpha_1, \alpha_2, ..., ...\alpha_n \in \mathbb{R}$ such that

$$x = \sum_{i=1}^{n} \alpha_i x_i$$

Prove that α_i is unique for all i. (Hint: use Proposition 1).

2) Prove or disprove the following statement: any vector space X has a unique basis.

3) Prove that if X is an n-dimensional linear space, then any set $S \subset X$ of $n+1$ elements is linearly dependent. (Hint: use Propositions 1 and 5).