

# An Introduction to Mathematics for Political Science

## Problem Set 1

You are encouraged to work in groups and actively participate on the course discussion page. Submitted solutions must be your individual work. Do not use a calculator or search for solutions. Show all of your work. Submit typed solutions or scans of handwritten solutions as a PDF. Note that starting with problem set 3 all solutions must be written in LaTeX.

### Set Basics

1. Let  $A = (40, 60)$  and  $B = [50, 70]$ .
  - a) Is  $A \subset B$ ,  $B \subset A$ , both, or neither?
  - b) What is  $A \cup B$ ?
  - c) What is  $A \cap B$ ?
  - d) Write three elements of the Cartesian product  $A \times B$ .
2. Identify whether the following sets are (a) open, closed, or neither; (b) bounded; (c) compact; (d) convex:
  - a)  $(0, 1)$
  - b)  $[0, 1]$
  - c)  $(0, 1]$
  - d)  $[0, \infty)$
  - e)  $(0, \infty)$

f)  $[0, 1] \cup [2, 3]$

g)  $[0, 1] \times [0, 1]$

h)  $(0, 3] \cap [1, 4]$

i)  $[0, 5] \setminus \{1, 2\}$

3. Express the following sentences in mathematical notation:

a)  $A$  is the set of all real numbers less than or equal to seven, excluding zero and four.

b)  $B$  is the intersection of the natural numbers and the real numbers between  $\pi$  and 30.5.

c) For all epsilon greater than zero, there exists a delta greater than zero.

d) The set of all even integers between 5 and 21.

## Algebra

4. Simplify into one term or evaluate the following:

a)  $y \cdot y \cdot y \cdot y$

b)  $(-a)(-b)^3 - b^2 + a^3$

c)  $(4b + 2)(a - 5)$

d)  $\frac{5!}{2!}$

e)  $\sum_{i=1}^3 \left(\frac{1}{3}\right)^i$

f)  $\sum_{i=2}^5 2^i$

g)  $\prod_{i=1}^3 \left(\frac{1}{3}\right)^i$

h)  $\frac{48}{4} - 6 \cdot 9$

i)  $(3^3 + (-5)) \cdot 3 - (-7)$

j)  $\left[6 + \left(\frac{-66}{11}\right)\right] \cdot (-2)^3$

k)  $\frac{y-11}{5} + \frac{y+12}{3}$

5. Solve the following for  $x$ :

a)  $5(-3x - 2) - (2x - 3) = -4(4x + 5) + 13$

b)  $8x^2 = 15 - 14x$

c)  $x^2 - 3x + 4 = 2(x - 1)$

d)  $-6 > 5x + 5 + 4$

e)  $-2(x + 1) + 4 < 10$

f)  $2\ln(2x + 1) - 10 = 0$

g)  $3^x e^{4x} = e^7$

## Functions

6) Find the image/range of  $x^2$  on the domain  $[-3, 3]$ .

7) Let  $f(x) = x^2 - 4x + 2$  and  $g(x) = 3x - 7$ . Find (a)  $f(x) - g(x)$ ; (b)  $f(x) + g(x)$ ; (c)  $f(g(x))$ ; and (d)  $g(f(x))$ . Evaluate each for  $x = 2$ .

8) Identify whether each of the following functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  is (a) surjective/onto; (b) injective/one-to-one; (c) bijective.

a)  $f(x) = x^2$

b)  $f(x) = x^3 - x$

c)  $f(x) = e^x$

d)  $f(x) = x^3$

9) Find an equation for the inverse for each of the following functions:

a)  $f(x) = (5x - 1)^3$

b)  $f(x) = \frac{x+4}{3x-5}$

c)  $f(x) = e^{5x-1}$

10) Evaluate each of the following limits or show that they do not exist:

a)  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x^2 + x - 30}$

b)  $\lim_{x \rightarrow -1} \frac{x^3}{(x+1)^2}$

c)  $\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{|x - 2|}$

d)  $\lim_{x \rightarrow \infty} \frac{x^2 - 1}{2x^2 + 1}$

e)  $\lim_{x \rightarrow \infty} \left( \frac{x^3}{x^2 + 2} - x \right)$

$$\text{f) } \lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1}}{x}$$

11) Determine whether the following functions are continuous at the specified value. Use limits where appropriate. You may find sketching the graph of each function to be helpful.

$$\text{a) } f(x) = \frac{x^2+1}{x^3+1} \text{ at } x = -1$$

$$\text{b) } f(x) = \begin{cases} 3x - 5 & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases} \text{ at } x = 1$$

$$\text{c) } f(x) = \begin{cases} \frac{x-6}{x-3} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \sqrt{4+x^2} & \text{if } x > 0 \end{cases} \text{ at } x = 0$$