

An Introduction to Mathematics For Political Science

Problem Set 3

You are encouraged to work in groups and actively participate on the course discussion page. Submitted solutions must be your individual work. Do not use a calculator or search for solutions. Show all of your work. Starting this week all solutions must be written in LaTeX.

Integration

1. Evaluate the following:

a) $\int x^2 + 2x + 2 \, dx$

$$\frac{x^3}{3} + x^2 + 2x + C$$

b) $\int x^7 + x^{-3} - \frac{1}{x} \, dx$

$$\frac{x^8}{8} - \frac{1}{2x^2} - \ln(|x|) + C$$

c) $\int_4^9 \frac{1}{x\sqrt{x}} \, dx$

$$\int_4^9 x^{-3/2} = \left. \frac{1}{-1/2} x^{-1/2} \right]_4^9 = -2(9)^{-1/2} - (-2(4)^{-1/2}) = -2/3 + 2/2 = 1/3$$

d) $\int e^{-x/3} \, dx$

$$\frac{1}{-1/3} e^{-x/3} + C = -3e^{-x/3} + C$$

e) $\int e^{2x} - 2e^x - 2ex^2 \, dx$

$$\frac{e^{2x}}{2} - 2e^x - \frac{2ex^3}{3} + C$$

f) $\int_{-e^2}^{-1} \frac{4}{x} dx$

$$\int_{-e^2}^{-1} \frac{4}{x} dx = 4 \ln |x| \Big|_{-e^2}^{-1} = 4 \ln |-1| - 4 \ln |-e^2| = 4 \ln(1) - 4 \ln(e^2) = 8$$

g) $\int_0^2 2x^2 + 3x + 1 dx$

The indefinite integral is $\frac{x(4x^2+9x+6)}{6} + C$. Now evaluate $\frac{2(4(2)^2+9(2)+6)}{6} - \frac{0(4(0)^2+9(0)+6)}{6} = \frac{40}{3}$

h) $\int_{-a}^a x^5 dx$

$$\int_{-a}^a x^5 = \left. \frac{x^6}{6} \right|_{-a}^a = \frac{a^6}{6} - \frac{(-a)^6}{6} = \frac{a^6}{6} - \frac{a^6}{6} = 0$$

i) $\int e^{\sqrt{x}} dx$

Let $u = \sqrt{x}$. $\frac{du}{dx} = \frac{1}{2\sqrt{x}} \rightarrow dx = 2\sqrt{x} du = 2u du$. Substitute u and dx into the original integral to get $\int 2ue^u du = 2 \int ue^u du$. Now integrate by parts: $\int fg' = fg - \int f'g$ where we let $f = u$ and $g' = e^u$. Note that $f' = \frac{\partial}{\partial u} u = 1$. To find g , take the antiderivative of e^u which is simply e^u . Now we have $\int ue^u du = ue^u - \int e^u du = ue^u - e^u$. Finish by plugging everything back into $\int 2ue^u du$:

$$\int 2ue^u du = 2ue^u - 2e^u + C = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C = 2(\sqrt{x} - 1)e^{\sqrt{x}} + C$$

j) $\int \frac{e^{2x}}{2} dx$

Let $u = 2x$. $\frac{du}{dx} = 2 \rightarrow dx = \frac{1}{2} du$. Substitution gives us $\int \frac{e^{2x}}{2} dx = \frac{1}{4} \int e^u du = \frac{e^u}{4}$ Now finish by substituting $u = 2x$ and adding a constant: $\frac{e^{2x}}{4} + C$.

k) $6 \int xe^{2x} dx$

We need to integrate by parts. Let $f = x$ and $g' = e^{2x}$ so that $f' = 1$ and $g = \frac{e^{2x}}{2}$. Now we have $fg - \int f'g = \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2} dx$. In the previous problem we found that $\int \frac{e^{2x}}{2} dx = \frac{e^{2x}}{4} + C$ so the solution is $6\left(\frac{xe^{2x}}{2} - \frac{e^{2x}}{4}\right) = 3xe^{2x} - \frac{3e^{2x}}{2} + C$.

l) $\int \frac{(6x^2+5)e^{2x}}{2} dx$

Note that $\frac{1}{2} \int (6x^2+5)e^{2x} dx$. Integrate by parts: $f = 6x^2+5$, $g' = e^{2x}$, $f' = 12x$, $g = \frac{e^{2x}}{2}$.

$$fg - \int f'g = \frac{(6x^2+5)e^{2x}}{2} - 6 \int xe^{2x} dx$$

In the previous problem we found that $6 \int xe^{2x} dx = 3xe^{2x} - \frac{3e^{2x}}{2} + C$ so our solution is

$$\frac{1}{2} \left(\frac{(6x^2+5)e^{2x}}{2} - 3xe^{2x} + \frac{3e^{2x}}{2} \right) = \frac{(6x^2+5)e^{2x}}{4} - \frac{3xe^{2x}}{2} + \frac{3e^{2x}}{4} + C$$

m) $\int (2x^3 + 5x + 1)e^{2x} dx$

Use integration by parts: $\int fg' = fg - \int f'g$. Let $f = 2x^3 + 5x + 1$ and $g' = e^{2x}$.

$f' = 6x^2 + 5$ and $g = \frac{e^{2x}}{2}$. Now we just follow the formula:

$$\int f g' = f g - \int f' g = \frac{(2x^3+5x+1)e^{2x}}{2} - \int \frac{(6x^2+5)e^{2x}}{2} dx.$$

In the previous problem we found that

$$\int \frac{(6x^2+5)e^{2x}}{2} dx = \frac{(6x^2+5)e^{2x}}{4} - \frac{3xe^{2x}}{2} + \frac{3e^{2x}}{4} + C$$

so we conclude that

$$\int (2x^3 + 5x + 1)e^{2x} dx = \frac{(2x^3+5x+1)e^{2x}}{2} - \frac{(6x^2+5)e^{2x}}{4} + \frac{3xe^{2x}}{2} - \frac{3e^{2x}}{4} + C$$

which simplifies to

$$\frac{(2x^3-3x^2+8x-3)e^{2x}}{2} + C$$